

THE EFFECTS OF EXTERNAL FORCING, DISSIPATION AND NONLINEARITY ON THE SOLUTIONS OF ATMOSPHERIC EQUATIONS*

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ABSTRACT

Based on the primitive equations of the atmosphere, we study the effects of external forcing, dissipation and nonlinearity on the solutions of stationary motion and non-stationary motion. The results show that the asymptotic behavior of solutions of the forced dissipative nonlinear system is essentially different from that of the adiabatic non-dissipative system, the adiabatic dissipative system, the diabatic non-dissipative system and the diabatic dissipative linear system, and that the joint action of external forcing, dissipation and nonlinearity is the source of multiple equilibria. From this we can conclude that the important actions of diabatic heating and dissipation must be considered in the models of the long-term weather and the climate.

Key words: external forcing, dissipation, nonlinearity, operator equation, asymptotic behavior, attractor

1. INTRODUCTION

The atmosphere is a forced dissipative nonlinear system. There are two main difficulties to study it. One is the complexity of interactive process associated with diabatic heating and friction, the other is the complexity of nonlinearity (Chou 1986; 1991; 1995). To avoid them, the traditional dynamical meteorology is limited to the linear theory or the nonlinear conservative theory. It is clear that these theories are successful in the explanations of short-term evolution and the short-term numerical forecast for synoptic scale atmosphere. Nevertheless, they can not be applied to explain the long-term evolution of planetary circulation, nor can they be successfully used to simulate and forecast the long-term behavior of the atmosphere. It is because of the problems neglected as follows: the different effects of external forcing, dissipation and nonlinearity on the asymptotic behavior of the solutions, the irreversible process of the system, the interaction between the irreversible process and the nonlinear reversible process, and the important actions of diabatic heating and dissipation, etc. Chou (1986; 1995) studied them using the operator equation of large-scale atmospheric motion and the properties of operators, pointed out some differences in the asymptotic behavior of solutions among the forced dissipative nonlinear system, the adiabatic non-dissipative system and the forced dissipative linear

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system. In this paper, further researches are carried out on the effects of external forcing, dissipation and nonlinearity on the solutions of atmospheric equations using the completed primitive nonlinear operator equation of the atmosphere.

II. OPERATOR EQUATION. PROPERTIES OF OPERATORS AND PROBLEM DESCRIPTION

In the spherical coordinate system $(\lambda, \theta, r; t)$ (λ is the longitude, θ the colatitude, r the geocentric distance), the completed primitive equations of the atmosphere (Zeng 1979; Li and Chou 1996, 1997b) can be written as the following operator equations:

$$\begin{cases} \frac{\partial \varphi}{\partial t} + (N(\varphi) + L(\varphi))\varphi = \xi(\varphi), \\ \varphi|_{t=0} = \varphi_0, \end{cases} \quad (1)$$

where $\varphi = (\tilde{V}_\lambda, \tilde{V}_\theta, \tilde{V}_r, \tilde{\rho}, \tilde{T})'$, $\tilde{V}_\lambda = \rho^* V_\lambda^*$, $\tilde{V}_\theta = \rho^* V_\theta^*$, $\tilde{V}_r = \rho^* V_r^*$, $\tilde{\rho} = \sqrt{\Phi} \rho^*$, $\tilde{T} = \rho^* T^*$, $V_\lambda^* = V_\lambda / \sqrt{2}$, $V_\theta^* = V_\theta / \sqrt{2}$, $V_r^* = V_r / \sqrt{2}$, $\rho^* = \sqrt{\rho}$, $T^* = \sqrt{C_v T}$, $\Phi = gr$,

$$N(\varphi) = \begin{vmatrix} \mathcal{L} & F_1 & F_2 & 0 & \frac{1}{\rho^* r \sin \theta} \frac{\partial}{\partial \lambda} G \\ -F_1 & \mathcal{L} & \frac{V_\theta}{r} & 0 & \frac{1}{\rho^* r} \frac{\partial}{\partial \theta} G \\ -F_2 & -\frac{V_\theta}{r} & \mathcal{L} & \frac{g}{\sqrt{2\Phi}} & \frac{1}{\rho^*} \frac{\partial}{\partial r} G \\ 0 & 0 & -\frac{g}{\sqrt{2\Phi}} & \mathcal{L} & 0 \\ \frac{G}{r \sin \theta} \frac{\partial}{\partial \lambda} \frac{1}{\rho^*} & \frac{G}{r \sin \theta} \frac{\partial}{\partial \theta} \frac{\sin \theta}{\rho^*} & \frac{G}{r^2} \frac{\partial}{\partial r} \frac{r^2}{\rho^*} & 0 & \mathcal{L} \end{vmatrix},$$

$$L(\varphi) = \begin{vmatrix} -\frac{\mu_1}{3\rho^* r \sin \theta} \frac{\partial}{\partial \lambda} l_1 - \mu_1 l_4 - l & -\frac{\mu_1}{3\rho^* r \sin \theta} \frac{\partial}{\partial \lambda} l_2 & -\frac{\mu_1}{3\rho^* r \sin \theta} \frac{\partial}{\partial \lambda} l_3 & 0 & 0 \\ -\frac{\mu_1}{3\rho^* r} \frac{\partial}{\partial \theta} l_1 & -\frac{\mu_1}{3\rho^* r} \frac{\partial}{\partial \theta} l_2 - \mu_1 l_4 - l & -\frac{\mu_1}{3\rho^* r} \frac{\partial}{\partial \theta} l_3 & 0 & 0 \\ -\frac{\mu_1}{3\rho^*} \frac{\partial}{\partial r} l_1 & -\frac{\mu_1}{3\rho^*} \frac{\partial}{\partial r} l_2 & -\frac{\mu_1}{3\rho^*} \frac{\partial}{\partial r} l_3 - \mu_1 l_4 - l & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -l_5 \end{vmatrix},$$

$$\xi(\varphi) = (0, 0, 0, 0, \varepsilon/2\tilde{T})',$$

where

$$\mathcal{L} = (\Pi + \Lambda)/2,$$

$$\Pi = \frac{1}{r \sin \theta} \frac{\partial}{\partial \lambda} V_\lambda + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} V_\theta \sin \theta + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 V_r,$$

$$\Lambda = \frac{V_\lambda}{r \sin \theta} \frac{\partial}{\partial \lambda} + \frac{V_\theta}{r} \frac{\partial}{\partial \theta} + V_r \frac{\partial}{\partial r},$$

$$F_1 = 2\Omega \cos\theta + V_\lambda \operatorname{ctg}\theta/r,$$

$$F_2 = 2\Omega \sin\theta + V_\lambda/r,$$

$$G = R\tilde{T}/\sqrt{2}C_v,$$

$$l = \frac{1}{\rho^*} \left[\frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \lambda} k_\lambda \frac{\partial}{\partial \lambda} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} k_\theta \sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial}{\partial r} k_r r \frac{\partial}{\partial r} \right] \frac{1}{\rho^*},$$

$$l_1 = \frac{1}{r \sin\theta} \frac{\partial}{\partial \lambda} \frac{1}{\rho^*},$$

$$l_2 = \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \frac{\sin\theta}{\rho^*},$$

$$l_3 = \frac{1}{r^2} \frac{\partial}{\partial r} \frac{r^2}{\rho^*},$$

$$l_4 = \frac{1}{\rho^*} \Delta \frac{1}{\rho^*},$$

$$l_5 = \frac{1}{\rho^*} \left[\frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \lambda} K_\lambda \frac{\partial}{\partial \lambda} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} K_\theta \sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial}{\partial r} K_r r \frac{\partial}{\partial r} \right] \frac{1}{\rho^*},$$

$$\Delta = \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \lambda^2} + \frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \theta} \sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r},$$

μ_1 is the molecular viscosity coefficient, k_i ($i=\lambda, \theta, r$) the turbulent viscosity coefficient, K_i ($i=\lambda, \theta, r$) the turbulent thermal conductivity, ϵ the diabatic heating, all other notations are usual meteorologically. The domain of solutions $\Omega = S^2 \times (a, r_\infty)$ with $a < r_\infty < \infty$ (a is radius of the earth, r_∞ a certain large number). The boundary value conditions are given below:

On the earth's surface $r=a$,

$$V_\lambda = V_\theta = V_r = 0, \quad (3)$$

$$\frac{\partial T}{\partial r} = \alpha_s (T - T_s), \quad (4)$$

where T_s is the temperature on the surface of the earth, α_s is a positively constant related to turbulent thermal conductivity.

At the top of atmosphere $r=r_\infty$,

$$(\rho V_\lambda^2, \rho V_\theta^2, \rho V_r^2, \rho T, \rho \Phi) = 0, \quad (5)$$

$$\frac{\partial V_\lambda}{\partial r} = \frac{\partial V_\theta}{\partial r} = 0, \quad V_r = 0, \quad \frac{\partial T}{\partial r} = 0. \quad (6)$$

For convenience, we only consider the homogeneous boundary value condition of Eq. (4) given by

$$\frac{\partial T}{\partial r} = \alpha_s T. \quad (7)$$

The operator equation (1) concentrically reflects the dissipative property of the atmosphere under the thermal forcing. The operator $L(\varphi)$ represents dissipation, $\xi(\varphi)$ the external forcing, $N(\varphi)$ the actions of the nonlinear advection, the Coriolis force, the sphericity and the gravity etc. We study in this paper the effect of external forcing, dissipation and nonlinearity on the solutions of atmospheric equations, and the subject will be discussed from the following five points:

(1) The forced dissipative nonlinear system of the atmosphere, namely the asymptotic property of solutions of the operator equation (1);

(2) The adiabatic non-dissipative system, namely the equation:

$$\frac{\partial \varphi}{\partial t} + N(\varphi)\varphi = 0; \quad (8)$$

(3) The adiabatic dissipative system, namely the equation

$$\frac{\partial \varphi}{\partial t} + N(\varphi)\varphi + L(\varphi)\varphi = 0 \quad (9)$$

and the corresponding stationary equation

$$N(\varphi)\varphi + L(\varphi)\varphi = 0; \quad (10)$$

(4) The forced non-dissipative system, namely the equation

$$\frac{\partial \varphi}{\partial t} + N(\varphi)\varphi = \xi(\varphi) \quad (11)$$

and the corresponding stationary equation

$$N(\varphi)\varphi = \xi(\varphi); \quad (12)$$

(5) The forced dissipative linear system, namely the equation

$$\frac{\partial \varphi}{\partial t} + N(\bar{\varphi})\varphi + L(\bar{\varphi})\varphi = \xi(\bar{\varphi}) \quad (13)$$

and the corresponding stationary equation

$$N(\bar{\varphi})\varphi + L(\bar{\varphi})\varphi = \xi(\bar{\varphi}), \quad (14)$$

where $\bar{\varphi}$ is a known function independent of φ .

The existence of the solutions will not be discussed in this paper.

Let $H_0(\Omega)$ be the complete space with the interior product and the norm as follows:

$$(\varphi_1, \varphi_2) = \int_{\Omega} \varphi_1' \varphi_2' d\Omega = \int_a^{r_{\infty}} \int_0^{\pi} \int_0^{2\pi} \varphi_1' \varphi_2' r^2 \sin\theta d\lambda d\theta dr, \quad (15)$$

$$\|\varphi\|_0 = (\varphi, \varphi)^{1/2}, \quad (16)$$

where $\varphi = (\tilde{V}_{i\lambda}, \tilde{V}_{i\theta}, \tilde{V}_{ir}, \tilde{\rho}_i, \tilde{T}_i)'$, $i=1,2$. Apparently, $H_0(\Omega)$ is a Hilbert space. Let $\|\cdot\|$ always be the norms in $L^2(\Omega)$. Obviously,

$$\|\varphi\|_0 = (\|\tilde{V}_{\lambda}\|^2 + \|\tilde{V}_{\theta}\|^2 + \|\tilde{V}_r\|^2 + \|\tilde{\rho}\|^2 + \|\tilde{T}\|^2)^{1/2}, \quad (17)$$

$\forall \varphi = (\tilde{V}_{\lambda}, \tilde{V}_{\theta}, \tilde{V}_r, \tilde{\rho}, \tilde{T})' \in H_0(\Omega)$. Then we have

Lemma 1. $L(\varphi)$ is a positively definite self-adjoint operator, $N(\varphi)$ an anti-adjoint operator.

Removing $1/\rho^*$ from the operator $L(\varphi)$ and let the rest be the operator \tilde{L} . It is clear that \tilde{L} is a linear operator.

Lemma 2.

$$(L(\varphi)\varphi, \psi) = (\tilde{L}\psi, \psi), \quad (18)$$

$\forall \varphi = (\tilde{V}_{\lambda}, \tilde{V}_{\theta}, \tilde{V}_r, \tilde{\rho}, \tilde{T})'$, $\psi = (V_{\lambda}^*, V_{\theta}^*, V_r^*, \rho^*, T^*)' \in H_0(\Omega)$.

In $H_0(\Omega)$, it is easy to see that we can use the following equivalent norm

$$\|\varphi\|_0 = (\|\tilde{V}_{\lambda}\|^2 + \|\tilde{V}_{\theta}\|^2 + \|\tilde{V}_r\|^2 + \|\rho^*\|^2 + \|\tilde{T}\|^2)^{1/2}. \quad (19)$$

Let

$$|\varphi|_0 = (\|\tilde{V}_{\lambda}\|^2 + \|\tilde{V}_{\theta}\|^2 + \|\tilde{V}_r\|^2 + \|\tilde{T}\|^2)^{1/2}. \quad (20)$$

It is easy to see that

$$\|\varphi\|_0 = 0 \Leftrightarrow |\varphi|_0 = 0. \quad (21)$$

Let $H_1(\Omega)$ be the complete space with the norm

$$\|\psi\|_1 = (\|V_{\lambda}^*\|_{H^1}^2 + \|V_{\theta}^*\|_{H^1}^2 + \|V_r^*\|_{H^1}^2 + \|\rho^*\|^2 + \|T^*\|_{H^1}^2)^{1/2}, \quad (22)$$

where $\phi = (V_\lambda^*, V_\theta^*, V_r^*, \rho^*, T^*)'$, $\|\cdot\|_{H^1}$ takes $H^1(\Omega)$ -norm. Here $H^1(\Omega)$ is the standard Sobolev space. And let

$$|\phi|_1 = (\|V_\lambda^*\|_{H^1}^2 + \|V_\theta^*\|_{H^1}^2 + \|V_r^*\|_{H^1}^2 + \|T^*\|_{H^1}^2)^{1/2}. \quad (23)$$

Then we have

$$\|\phi\|_1 = 0 \Leftrightarrow |\phi|_1 = 0. \quad (24)$$

Lemma 3. There exists constant $C > 0$ such that

$$|\varphi|_0^2 \leq C |\phi|_0^2, \quad (25)$$

$\forall \varphi = (\tilde{V}_\lambda, \tilde{V}_\theta, \tilde{V}_r, \tilde{\rho}, \tilde{T})'$, $\phi = (V_\lambda^*, V_\theta^*, V_r^*, \rho^*, T^*)' \in H_0(\Omega)$.

Lemma 4. There exists constant $C_1 > 0$ such that

$$C_1 |\phi|_1^2 \leq (\tilde{L}\phi, \phi), \quad (26)$$

$\forall \phi = (V_\lambda^*, V_\theta^*, V_r^*, \rho^*, T^*)' \in H_1(\Omega)$.

III. FORCED DISSIPATIVE NONLINEAR SYSTEM

For the forced dissipative nonlinear system of the atmosphere, we have studied the properties of its solutions in detail (Li and Chou 1997a, b; Wang et al. 1990; Lions et al. 1992). The main results now may be summarized as follows.

Theorem 1. Any solution φ of the operator equations (1), (2) satisfies

$$\|\varphi\|_0^2 \leq \{\|\varphi_0\|_0^2 + 2 \int_0^t e^{-\tilde{C}_1 s} (C_m + |\varepsilon(s)|) ds\} e^{-\tilde{C}_1 t}, \quad t \in [0, T], \text{ a.e.}, \quad (27)$$

\tilde{C} and C_m are the positively constants.

As the time $t \rightarrow \infty$, $\|\varphi_0\|_0^2 e^{-\tilde{C}_1 t} \rightarrow 0$. Eq. (27) shows that the atmosphere system has the characteristic of the decay of the effect of initial field, and the long-range evolution of the system will depend on the variation of the external forcing.

In reality, the external forcing should be the bounded; namely, $\varepsilon \in L^\infty(R_+)$, then let

$$M = |\varepsilon|_{0, \infty, R_+} = \text{ess sup}_{t \in R_+} |\varepsilon(t)|. \quad (28)$$

Furthermore, Eqs. (1) and (2) define the continuous mapping (i. e. the solution operator) $S(t): H_0 \rightarrow H_0$ such that $S(t)\varphi_0 = \varphi(t)$. We have

$$S(t)R = \{S(t)\varphi_0 | \forall \varphi_0 \in R \subset H_0\}. \quad (29)$$

It is easy to see that $S(t)$ is a semigroup.

Theorem 2. There exists a bounded absorbing set B_K in H_0 and a time $\tau(R)$ such that for any bounded set $R \subset H_0$

$$S(t)R \subset B_K, \quad t \geq \tau(R). \quad (30)$$

We define

$$A = \bigcap_{s \geq 0} \bigcup_{t \geq s} S(t)B_K \quad (31)$$

Then we have

Theorem 3. The set A satisfies

- (1) A is a bounded set in H_0 ;
- (2) A is a functional invariant set of the semigroup $S(t)$, i. e.,

$$S(t)A = A, \quad t \geq 0; \quad (32)$$

- (3) There exists open neighbourhood U of A such that for any $\varphi_0 \in U$ one has $S(t)\varphi_0 \rightarrow A$ as $t \rightarrow \infty$;

- (4) A uniformly attracts the set B_K ;
 (5) A is a global attractor of the semigroup $S(t)$.

The above conclusions show that the atmosphere system will trend towards the global attractor A as the time $t \rightarrow \infty$ (If A is a compact attractor and attracts any bounded set in H space, then A is called the global attractor of the semigroup $S(t)$); that is to say, the asymptotic behavior of its solutions shows itself on the structure of attractor. Moreover, the long-term evolution of the system depends on the state of the external forcing according to the results mentioned before. So, once the external forcings are fixed, the final state of the system will adjust itself to an attractor corresponding to the external forcings. In the physical sense, the behavior is just nonlinear adjustment to the external forcing. Besides, it can display chaotic phenomenon.

IV. ADIABATIC NON-DISSIPATIVE SYSTEM

Theorem 4. Eq. (8) has the property of the conservation of energy.

By making H_0 inner product with φ for the sides of Eq. (8) and using the anti-symmetric property of the operator $N(\varphi)$, we have

$$\frac{d}{dt} \|\varphi\|_0^2 = 0, \quad (33)$$

namely

$$\begin{aligned} \|\varphi\|_0^2 &= \int_{\Omega} \left(\frac{V_{\lambda}^2 + V_{\theta}^2 + V_r^2}{2} + \varphi + C_v T \right) \rho d\Omega \\ &= \|\varphi_0\|_0^2 \\ &= \text{const.} \end{aligned} \quad (34)$$

It follows that there is the conservation of energy if the atmosphere is regarded as an adiabatic non-dissipative system. The effect of initial value does not decay up to infinity. Once the initial value is fixed, the system will be limited to one and the same isoenergy sphere. There is no attractor in the system. Moreover, the volume of phase space of the conservative system is constant in motion according to the Liouville theorem. On the contrary, the phase volume of the dissipative system has the contracting property that leads the solution trajectory to an attractor. These are essential distinctions between the adiabatic non-dissipative system and the forced dissipative nonlinear system.

V. ADIABATIC DISSIPATIVE SYSTEM

First of all, for the stationary motion Eq. (10), by making H_0 inner product with φ for the sides of Eq. (10) and using the antisymmetric property of the operator $N(\varphi)$, we have

$$(L(\varphi)\varphi, \varphi) = 0. \quad (35)$$

By use of the property of the positively definite self-adjoint of the operator $L(\varphi)$, we get

$$\|\varphi\|_0 = 0. \quad (36)$$

So, we obtain

Theorem 5. There is unique zero solution of Eq. (10).

For the non-stationary motion Eq. (9), we have

Theorem 6. Any solution φ for Eqs. (9) and (2) satisfies

$$|\varphi|_0^2 \leq |\varphi_0|_0^2 e^{-\alpha t}, \quad t \in [0, T], \text{ a. e. }, \quad (37)$$

Moreover,

$$\lim_{t \rightarrow \infty} \varphi(t) = 0. \quad (38)$$

The theorems mentioned above show that the adiabatic dissipative system has some properties as follows: the effect of initial field is decayed, the energy dissipates as the time t increases, there exists unique final state that is not associated with the initial value, and the final state is just the stationary solution of the system. From this we can conclude that it will become a dead structure if the system with dissipation does not obtain the supplementary energy from outside.

VI. FORCED NON-DISSIPATIVE SYSTEM

The atmosphere must depend on absorbing continuously the energy from the sun in order to maintain its motion. Therefore, we claim the diabatic heating satisfying

$$\int_a \varepsilon d\Omega > 0. \quad (39)$$

That is to say, the system always obtains the energy from outside. So, by making H_0 inner product with φ for the sides of Eq. (12), we get

$$\int_a \varepsilon d\Omega \equiv 0. \quad (40)$$

Equation (40) is in contradiction with Eq. (39). Thus, we obtain

Theorem 7. There does not exist any solution for Eq. (12).

For the non-stationary motion Eq. (11), we have

Theorem 8. Any solution φ of the Eqs. (11) and (2) satisfies

$$\|\varphi\|^2 = \|\varphi_0\|^2 + 2 \int_0^t \int_a \varepsilon(t) d\Omega dt, \quad t \in [0, T]. \quad (41)$$

Moreover

$$\lim_{t \rightarrow \infty} \|\varphi\|^2 \rightarrow \infty. \quad (42)$$

The above results show that there does not exist equilibrium state (namely stationary solution) in the forced non-dissipative system, and that the effect of initial value is unchanged within the arbitrary limited time domain. Furthermore, the energy of the system trends towards divergence as the time increases. From this it follows that it is absolute unstable if the system with energy import has not dissipation.

VII. FORCED DISSIPATIVE LINEAR SYSTEM

Theorem 9. There is unique solution φ_∞ for Eq. (14).

Theorem 10. Any solution φ of Eqs. (13) and (2) satisfies

$$\|\varphi\|^2 \leq \left\{ \|\varphi_0\|^2 + \frac{1}{C^*} \int_0^t e^{\tilde{C}t} (C_m + \|\xi(\bar{\varphi})\|^2) dt \right\} e^{-\tilde{C}t}, \quad t \in [0, T]. \quad (43)$$

Moreover

$$\lim_{t \rightarrow \infty} \varphi(t) = \varphi_\infty, \quad (44)$$

where φ_∞ is the stationary solution of Eq. (13).

Proof. We only have to prove Eq. (44).

Let $\tilde{\varphi}(t) = \varphi(t) - \varphi_\infty$, then

$$\frac{\partial \varphi}{\partial t} + N(\bar{\varphi})\bar{\varphi} + L(\bar{\varphi})\bar{\varphi} = 0, \quad (45)$$

making H_0 inner product with φ , and using Lemma 1 we have

$$\frac{1}{2} \frac{d}{dt} \|\varphi\|_0^2 + (L(\bar{\varphi})\bar{\varphi}, \bar{\varphi}) = 0. \quad (46)$$

Then using the positively definite property of the operator L and Lemma 4 and $|\tilde{\varphi}|_0^2 \leq |\tilde{\varphi}|_1^2$ we get

$$\frac{d}{dt} \|\tilde{\varphi}\|_0^2 + C |\tilde{\varphi}|_1^2 \leq 0, \quad (47)$$

$$\frac{d}{dt} |\tilde{\varphi}|_0^2 + C |\tilde{\varphi}|_1^2 \leq 0, \quad (48)$$

where C is a positively constant. By use of Gronwall inequality, we obtain

$$|\tilde{\varphi}|_0^2 \leq |\tilde{\varphi}(0)|_0^2 e^{-Ct}, \quad (49)$$

where $\tilde{\varphi}(0) = \varphi(0) - \varphi_\infty$. Equation (49) implies

$$\lim_{t \rightarrow \infty} |\tilde{\varphi}|_0^2 = 0. \quad (50)$$

The proof is completed.

According to the above results, we may conclude that the forced dissipative linear system has unique stationary solution, and that the effect of initial value will decay as the time increases, and that there exists unique final state that is not associated with the initial value; i. e., the asymptotic behavior of solutions shows itself on the structure of stationary solution.

VIII. SUMMARY AND DISCUSSION

The results in this paper show that the asymptotic behavior of solutions of the forced dissipative nonlinear system is essentially different from that of the adiabatic non-dissipative system, the adiabatic dissipative system, the diabatic non-dissipative system and the diabatic dissipative linear system. The major conclusions of this study may be listed as follows:

(1) For the adiabatic non-dissipative system, the effect of initial value does not decay, and there does not exist attractor, and its motion is limited to one and the same isoenergy sphere, and the volume of phase space keeps constant in motion.

(2) For the adiabatic dissipative system, the forced non-stationary system and the forced dissipative linear system, the stationary solution is either unique or non-existent, and they have not chaotic phenomenon, and the asymptotic behavior of solutions trends either stationary solution or divergence, and the final state of solutions is not connected with the initial value.

(3) For the forced dissipative nonlinear system, the asymptotic behavior of solutions shows itself on the structure of complicated attractor; and the long-term behavior depends on the state of external forcing, in other words, the system has the nonlinear adjustment process to external forcing. The system has chaotic phenomenon. It is not only characterized by the decay of effect of initial value (outside the global absorbing set, or during trending towards the domain of attraction of a certain attractor), but it is also extremely sensitive to initial value. In the state space, the state which is not in the state of

attractor will rapidly evolve into the state of attractor. The existence of chaotic motion shows that the long-term behavior of the system is characterized by the probability. The joint action of diabatic heating, dissipation and nonlinearity is the source of multiple equilibria. Thus, the system is able to display the catastrophe of flow pattern under the external forcing of gradual change, and to obtain the deterministic aperiodic flow (turbulence or chaos) from the deterministic physical law, and to form the motion types of strange attractor.

On the whole, the long-term behavior of the forced dissipative nonlinear system is essentially different from that of the adiabatic non-dissipative system, the adiabatic dissipative system, the diabatic non-dissipative system and the forced dissipative nonlinear system. For the long-term weather and the climate, we have to consider the important effects of diabatic heating and friction and treat them as the forced dissipative nonlinear system and build the forced dissipative nonlinear models. If not, we cannot obtain the convincing, reliable numerical forecast for them.

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